

INSTABILITY OF A LAYER OF LIGHTLY IONIZED PLASMA IN CROSSED
ELECTRIC AND MAGNETIC FIELDS

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In a lightly ionized plasma, charged-particle drift due to collisions with neutral atoms occurs at different velocities:

$$v_{Ea} = \mp \frac{b_a E}{1 + (\omega_a \tau_a)^2}, \quad v_{\perp a} = \frac{b_a E (\omega_a \tau_a)}{1 + (\omega_a \tau_a)^2}$$

$$\left(b_a = \frac{|e| \tau_a}{m_a}, \quad \omega_a = \frac{|e| \tau_a}{m_a} \right),$$

where b_a is the mobility of particles of the type a ; ω_a is the Larmor frequency; the upper sign refers to electrons and the lower sign to ions.

A difference in the charged-particle drift velocities can cause instability of an inhomogeneous lightly ionized plasma.

Let us consider the following example. Assume that in the initial state of the plasma there is a concentration gradient along the x -axis, that the external electric field is directed along the x -axis, and that the magnetic field coincides with the z -axis. In this system, under the influence of a Lorentz force the charged particles will move in a direction opposite to the y -axis. Since electrons have a higher velocity than ions, an electric field is induced in this direction. This electric field, together with the magnetic field, causes particle drift in the negative direction of the x -axis. Consequently, if the concentration gradient in the initial state is directed opposite to the x -axis this state cannot be stable.

Instability of this kind has been examined by Simon [1]. On the basis of studies by Kadomtsev and Nedospasov [2], as well as by Rosenbluth and Longmire [3], Simon developed a theory of instability of a lightly ionized plasma in crossed fields with an inhomogeneous density distribution in the direction of the external electric field. Somewhat later, Simon's theory was developed [4].

In devices with inhomogeneous plasma flow in which the plasma (conducting) layers alternate with nonconducting layers, the external electric field and concentration are normal to one another. We shall bear this case in mind below and shall examine the instability of a lightly ionized plasma in crossed fields when the concentration inhomogeneity is in a direction perpendicular to the external electric field.

§1. Let us assume that the plasma is inhomogeneous along the x -axis, that the external electric field is homogeneous and directed along the y -axis, and that the external magnetic field is homogeneous and has a z -component. The thickness of the plasma layer is $2a$, and the dimensions of the layer along the y - and z -axes are great enough that the boundary conditions can be ignored in these directions.

Let us consider oscillations at a frequency considerably lower than the Langmuir frequency. In addition, we assume that the thickness of the plasma layer is considerably greater than the Debye length. Therefore, the plasma can be considered quasineutral and the equation of charge conservation can be written as

$$\partial n / \partial t + \nabla n \mathbf{v}_e = Zn, \quad \partial n / \partial t + \nabla n \mathbf{v}_i = Zn. \quad (1.1)$$

Here, recombination is ignored, since it is assumed to be negligible in a lightly ionized plasma (note that recombination exerts a stabilizing influence on the

initial state of the plasma); n is the concentration; \mathbf{v}_e and \mathbf{v}_i are the directional electron and ion velocities; Z is the ionization frequency.

To determine the velocities \mathbf{v}_e and \mathbf{v}_i , we must use the equations of motion of the particles. Let the particle-velocity distribution be Maxwellian, let the electron T_e and ion T_i temperatures be constant (thermal diffusion is ignored), let the transport process be diffusional (the directional particle velocity is much less than the thermal), let the electric field have a potential nature (the induced magnetic field is ignored, in view of the low charged-particle concentration), let the collision frequency of ions with neutral atoms ν_{in} be much greater than the ion cyclotron frequency ω_i , and let the perturbation frequency ω and the charged-particle collision frequencies ν_{ei} and ν_{ie} be small in comparison with the frequencies ν_{en} and ν_{in} of collisions with neutral atoms. Thus, we can write

$$ne \nabla \psi - ne [\mathbf{v}_e \mathbf{B}] - T_e \nabla n - n m \nu_{en} \mathbf{v}_e = 0,$$

$$ne \nabla \psi + T_i \nabla n + n M \nu_{in} \mathbf{v}_i = 0. \quad (1.2)$$

Hence, the directional particle velocities are

$$v_{exi} = \beta_{xixj} \left(b_e \frac{\partial \psi}{\partial x_j} - \frac{D_e}{n} \frac{\partial n}{\partial x_j} \right),$$

$$(x_i = x, y, z; x_j = x, y, z), \quad (1.3)$$

$$\mathbf{v}_i = -b_i \nabla \psi - \frac{D_i}{n} \nabla n,$$

$$\left(b_e = \frac{|e| \tau_e}{m_e}, b_i = \frac{|e| \tau_i}{M}, D_e = \frac{T_e \tau_e}{m_e}, D_i = \frac{T_i \tau_i}{M} \right),$$

$$\beta_{xx} = \beta = [1 + (\omega_e \tau_e)^2]^{-1}, \beta_{xy} = -(\omega_e \tau_e) \beta, \beta_{xz} = 0,$$

$$\beta_{yx} = (\omega_e \tau_e) \beta, \beta_{yy} = \beta, \beta_{yz} = 0,$$

$$\beta_{zx} = 0, \beta_{zy} = 0, \beta_{zz} = 1. \quad (1.4)$$

In (1.3), a tensor notation is used for the sum of monomials with repeating subscripts; b_e and b_i are the electron and ion mobilities; D_e and D_i are the diffusion coefficients of these particles.

Along the y - and z -axes, the dimensions of the plasma layer are assumed to be rather large, so the perturbation of the concentration and of the electric field can be taken as $f(x) \exp i(k_y y + k_z z - \omega t)$. Therefore, in linearization of (1.1) we assume

$$n = n_0(x) + n_1(x) \exp i(k_y y + k_z z - \omega t),$$

$$\psi = \psi_0(x, y) + \psi_1(x) \exp i(k_y y + k_z z - \omega t),$$

$$\psi_0 = \psi_0(x, y) = \psi_0(x) + \psi_0(y), \quad (1.5)$$

$$\partial \psi_0 / \partial y = -E_0 = \text{const.}$$

Here E_0 is the external electric field. Considering (1.5), from (1.1) we obtain the continuity equations for

the initial state of the plasma

$$\beta \left[b_e \frac{d}{dx} \left(n_0 \frac{d\psi_0}{dx} \right) - D_e \frac{d^2 n_0}{dx^2} \right] + (\omega_e \tau_e) \beta b_e E_0 \frac{dn_0}{dx} = Zn_0, \\ - b_i \frac{d}{dx} \left(n_0 \frac{d\psi_0}{dx} \right) - D_i \frac{d^2 n_0}{dx^2} = Zn_0. \quad (1.6)$$

and the equations of charge conservation in the perturbations

$$-(Z + i\omega) n_1 + \beta \frac{d}{dx} \left[b_e \left(n_0 \frac{d\psi_1}{dx} + \frac{d\psi_0}{dx} n_1 \right) - D_e \frac{dn_1}{dx} \right] + \\ + (\omega_e \tau_e) \beta b_e \left[E_0 \frac{dn_1}{dx} + ik_y \left(\frac{d\psi_0}{dx} n_1 - \frac{dn_0}{dx} \psi_1 \right) \right] - \\ - \beta [k_y^2 (b_e n_0 \psi_1 - D_e n_1) + ik_y b_e E_0 n_1] - \\ - k_z^2 (b_e n_0 \psi_1 - D_e n_1) = 0 \\ -(Z + i\omega) n_1 - b_i \frac{d}{dx} \left(n_0 \frac{d\psi_1}{dx} + \frac{d\psi_0}{dx} n_1 \right) + \\ + k^2 (b_i n_0 \psi_1 + D_i n_1) + \\ + ik_y b_i E_0 n_1 - D_i \frac{d^2 n_1}{dx^2} = 0, \quad k^2 = k_y^2 + k_z^2. \quad (1.7)$$

The initial state of the plasma layer is determined by Eqs. (1.6). Various initial states are possible, depending on the boundary conditions. Let us assume that complete neutralization of the charged particles occurs at the boundaries of the plasma layer, i.e., $n_0 = 0$ when $x = \pm a$. A consequence of this assumption is singularity of the induced electric field at the points $x = \pm a$, but this most likely is formal in meaning.

We have

$$n_0 = N_0 e^{\nu x} \cos \nu x, \quad \nu = 1/2 \pi / a, \quad (1.8)$$

$$\frac{d\psi_0}{dx} = - \left(D_i + \frac{Z}{\rho^2 + \nu^2} \right) \frac{\rho}{b_i} + \left(D_i - \frac{Z}{\rho^2 + \nu^2} \right) \frac{\nu}{b_i} \operatorname{tg} \nu x, \quad (1.9)$$

$$\rho = \frac{\omega_e \tau_e}{2} \frac{b_i}{D_a} E_0, \quad Z = \frac{\beta b_e}{\beta b_e + b_i} D_a (\rho^2 + \nu^2), \quad (1.10)$$

$$D_a = \frac{1}{b_e} (b_i D_e + b_e D_i).$$

where D_a is the coefficient of ambipolar diffusion.

2. The problem of eigenvalues is raised in regard to the parameter $i\omega = i\omega_r + \alpha$ in (1.7). If there exists an eigenvalue with $\alpha < 0$, the initial state defined by (1.8) and (1.9) is unstable; otherwise, it is stable.

To solve this problem, we must find the perturbations n_1 and ψ_1 , but exact solution of Eqs. (1.7) is impossible, due to variability of the coefficients in them. We shall therefore use the Galerkin method. From the sequence of coordinate functions that satisfy the conditions of completeness, we take the first, thus limiting ourselves to seeking approximations for the first eigenvalue. We note that a similar simplification of the method was used earlier by Kadomtsev and Nedospasov [2] to study the instability of a positive column in a homogeneous longitudinal magnetic field with respect to helical perturbations. The results of their calculations of anomalous diffusion are in good agreement with the experimental data of Hoh and Lehnert [5].

Letting the perturbations n_1 and ψ_1 obey the boundary conditions $n_1 = \psi_1 = 0$ when $x = \pm a$, we take as the coordinate function $\cos \nu x$, i.e., we let

$$n_1 = N_1 \cos \nu x, \quad \psi_1 = \Psi_1 \cos \nu x, \quad (2.1)$$

where N_1 and Ψ_1 are complex constants.

Introducing (1.8), (1.9), and (2.1) into (1.7), with scalar multiplication by the coordinate function in the region $(-a < x < a)$, we obtain

$$\left\{ \beta k_\omega^2 D_e - D_a (\rho^2 + \nu^2) - \alpha - \right. \\ \left. - i \left[\omega_r + \frac{\beta}{\omega_e \tau_e} \frac{b_e}{b_i} k_y \rho (2D_a + (\omega_e \tau_e)^2 (D_a + D_i)) \right] \right\} N_1 - \\ - \left(F_2 + i \frac{2}{3} \omega_e \tau_e k_y \rho \right) \beta b_e \frac{n_0^*}{2} \Psi_1 = 0, \quad (2.2)$$

$$[D_i k^2 - D_a \rho^2 - \alpha - i (\omega_r - k_y b_i E_0)] N_1 + b_i \frac{n_0^*}{2} F_1 \Psi_1 = 0, \quad (2.3)$$

$$F_1 = \frac{\rho^2}{6} + \frac{\nu^2}{2} + k^2, \quad F_2 = \frac{\rho^2}{6} + \frac{\nu^2}{2} + k_\omega^2,$$

$$k_\omega^2 = k^2 + (\omega_e \tau_e k_z)^2, \quad n_0^* = \frac{2}{a} \int_{-a}^a n_0 \cos^2 \nu x dx.$$

Further, in (2.2) we ignore the term $D_a (\rho^2 + \nu^2)$, since it is smaller than the quantity $\beta k_\omega^2 D_e$ in the ratio $\beta b_i / b_e$. Then, with the condition of nontriviality of the solutions for N_1 and Ψ_1 , from (2.2) and (2.3) we obtain

$$\alpha = D_i k^2 - D_a \rho^2 + \\ + F_1 \frac{F_2 k_\omega^2 D_e b_i / b_e - 2/3 (k_y \rho)^2 [2D_a + (\omega_e \tau_e)^2 (D_a + D_i)]}{F_2^2 + (2/3 \omega_e \tau_e k_y \rho)^2}, \quad (2.4)$$

$$\omega_r = k_y b_i E_0 -$$

$$- k_y \rho F_1 \frac{2/3 \omega_e \tau_e k_\omega^2 D_e b_i / b_e + F_2 [2D_a + (\omega_e \tau_e)^2 (D_a + D_i)] / \omega_e \tau_e}{F_2^2 + (2/3 \omega_e \tau_e k_y \rho)^2}. \quad (2.5)$$

The initial state is unstable if $\alpha < 0$. Therefore, the instability criterion takes the form

$$\{ D_i k^2 [F_2^2 + (2/3 \omega_e \tau_e k_y \rho)^2] + F_1 F_2 k_\omega^2 D_e b_i / b_e \} < \rho^2 \{ D_a [F_2^2 + \\ + (2/3 \omega_e \tau_e k_y \rho)^2] + 2/3 k_y^2 [2D_a + (\omega_e \tau_e)^2 (D_a + D_i)] \}. \quad (2.6)$$

Let us make some estimates.

We denote by ν_0^2 the value of ν^2 at which the numerator of the fraction in (2.4) vanishes:

$$\nu_0^2 = \frac{2b_e}{k_\omega^2 b_i D_e} \left\{ \frac{2}{3} (k_y \rho)^2 [2D_a + \right. \\ \left. + (\omega_e \tau_e)^2 (D_a + D_i)] - k_\omega^2 \frac{b_i}{b_e} D_e \left(\frac{\rho^2}{6} + k_\omega^2 \right) \right\}.$$

Then the numerator of this fraction can be written as

$$M = 1/2 k_\omega^2 b_i D_e (\gamma - 1) \nu_0^2 / b_e \quad (\nu^2 = \gamma \nu_0^2).$$

Hence, it follows that:

1) If the wave numbers are such that $\gamma \geq 1$ (ν_0^2 cannot be less than zero), the increment will be no greater than $D_a \rho^2$, i.e.,

$$|\alpha| \leq (1/2 \omega_e \tau_e b_i E_0)^2 D_a^{-1}. \quad (2.7)$$

2) For other wave numbers

$$|\alpha| < (D_a \rho^2 + Q).$$

As Q we can take the upper limit of the fraction

$$\frac{2/3 (k_y \rho)^2 [2D_a + (\omega_e \tau_e)^2 (D_a + D_i)] F_1}{F_2^2 + (2/3 \omega_e \tau_e k_y \rho)^2}.$$

The quantity $2/3 k_y^2 F_1$ is not greater than the denominator of this fraction. Therefore,

$$Q = \rho^2 [2D_a + (\omega_e \tau_e)^2 (D_a + D_i)]$$

and, consequently,

$$|\alpha| < D_a^{-1} (\omega_e \tau_e b_i E_0)^2 [1 + (\omega_e \tau_e)^2]. \quad (2.8)$$

Comparing (2.7) and (2.8), we conclude that for any wave numbers the increment is less than

$$D_a^{-1} (\omega_e \tau_e \nu_{i0})^2 [1 + (\omega_e \tau_e)^2] \quad (\nu_{i0} = b_i E_0),$$

where v_{i0} is the directional (current) velocity of ions in the initial state of the plasma.

In (2.5), the fraction is less than ω_0 :

$$\begin{aligned}\omega_0 &= k_y \rho \left\{ \frac{b_i}{b_e} \omega_e \tau_e D_e + \frac{1}{\omega_e \tau_e} [2D_a + (\omega_e \tau_e)^2 (D_a + D_i)] \right\} = \\ &= k_y b_i E_0 [1 + (\omega_e \tau_e)^2].\end{aligned}$$

Therefore, for any wave numbers

$$|\omega_r| < k_y v_{i0} [1 + (\omega_e \tau_e)^2].$$

The value ω_r can be positive as well as negative. This indicates that forward and backward waves can exist.

As ν^2 increases, the positive part of the fraction in (2.4) decreases less markedly than does the negative part. Therefore, thin plasma layers must be less unstable than thick ones.

3. Let us consider the charged-particle balance in the plasma layer. In the initial state, the charged-particle diffusion currents that are directed along the y - and z -axes are not functions of the corresponding coordinates. Therefore, these currents cannot show up in the particle balance. The particle balance in the layer is determined by the diffusion currents that act along the x -axis and by ionization.

Along the x -axis, the particle currents are caused by the induced electric field and by the concentration gradient, and in the case of electrons, by the external electric field as well. The particle-drift velocities in this direction are the same, and the diffusion conditions are ambipolar and are maintained by the electric fields. The corresponding electron and ion currents Γ_{0x}^e and Γ_{0x}^i are determined by the relation

$$\Gamma_{0x}^e = \Gamma_{0x}^i = \frac{\beta b_e}{\beta b_e + b_i} D_a N_0 e^{\alpha x} (\rho \cos \nu x + \nu \sin \nu x).$$

Here, $\rho \cos \nu l + \nu \sin \nu l = 0$; to the left of the point $x = l$ these currents are directed opposite to the x -axis, and to the right of this point—along the x -axis. The total number of particles carried away by each of the currents Γ_{0x}^e and Γ_{0x}^i per unit time from a volume of $2a \times 1 \times 1$ is

$$2 \frac{\beta b_e}{\beta b_e + b_i} \nu N_0 \operatorname{ch} \rho a.$$

The same number of particles of each kind are produced per unit time in this volume by ionization. Thus, the particle balance in the layer is maintained, and this allows the initial state to be considered an equilibrium state.

To consider the particle balance in a perturbed state, we use the ion-continuity equation in perturbations

$$\begin{aligned}i\omega_r + \alpha n_1 &= \\ &= -b_i \left[\frac{\partial n_0}{\partial x} \frac{\partial \psi_0}{\partial x} + n_0 \Delta \psi_1 + \frac{\partial}{\partial x} \left(n_1 \frac{\partial \psi_0}{\partial x} \right) - E_0 \frac{\partial n_1}{\partial y} \right] - \\ &\quad - D_i \Delta n_1 - Z n_1.\end{aligned}\quad (3.1)$$

Here, n_1 and ψ_1 are given by relations (2.1), in which the following relationship exists between N_1 and Ψ_1 (see (2.3)):

$$\Psi_1 = \kappa N_1 e^{j\delta}, \quad \operatorname{tg} \delta = \frac{\omega_r - k_y b_i E_0}{\alpha + D_a \rho^2 - D_i k^2}, \quad \kappa = \operatorname{const}.$$

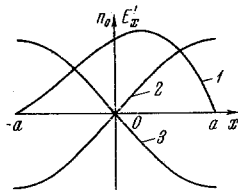


Fig. 1

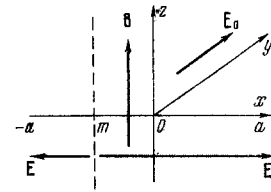


Fig. 2

Adhering, as before, to the ideas of the Galerkin method, we use orthogonality condition (3.1) with respect to the balance function $\cos \nu x$. We introduce

$$\{f\}^* = \int_{-a}^a f \cos \nu x dx.$$

Then we obtain

$$\{n_1\}^* = N_1 a, \quad \{\Delta n_1\}^* = -(\nu^2 + k^2) N_1 a, \quad (3.2)$$

$$\left\{ \frac{\partial}{\partial x} \left(n_1 \frac{\partial \psi_0}{\partial x} \right) \right\}^* = -\frac{\nu^2}{b_i} D_e N_1 a, \quad \Theta = -\frac{b_i}{2} F_1 \{n_0\}^* \kappa N_1 a \cos \delta,$$

$$\Theta = \operatorname{Re} \left(\left\{ \frac{\partial n_0}{\partial x} \frac{\partial \psi_1}{\partial x} \right\}^* + \{n_0 \Delta \psi_1\}^* \right). \quad (3.3)$$

Now it is easy to establish the origin of the individual terms in (2.4). The first two terms in (2.4) reflect the effect of ionization and of variation in the diffusion currents governed by the perturbed-concentration gradient and by the effect on this concentration of the electric field induced in the initial state,

$$-Z \{n_1\}^* - D_i \{\Delta n_1\}^* - b_i \left\{ \frac{\partial}{\partial x} \left(n_1 \frac{\partial \psi_0}{\partial x} \right) \right\}^* = D_i k^2 - D_a \rho^2,$$

since $Z \approx D_a (\rho^2 + \nu^2)$. Here, this current variation, as opposed to ionization, exerts a stabilizing influence on the initial state of the plasma layer. The role of the term Θ depends on the sign of $\cos \delta$. When $\delta > 0$, which means $(\alpha + D_a \rho^2 - D_i k^2) > 0$, this term is stabilizing; when $(\alpha + D_a \rho^2 - D_i k^2) < 0$, it is destabilizing. If we compare the real part of (3.1) with (2.4), we find that the condition $\cos \delta \gtrsim 0$ is equivalent to the following:

$$F_2 k_\omega^2 D_e b_i / b_e \gtrsim 2/3 (k_y \rho)^2 [2D_a + (\omega_e \tau_e)^2 (D_a + D_i)],$$

and the quantity Θ , which is governed by the effect of the perturbed electric field on the equilibrium concentration of ions, is adequate for the third term (fraction) in (2.4).

The role of the perturbed electric field is shown graphically in Fig. 1. In this figure, curve 1 represents the equilibrium-concentration distribution, and curves 2 and 3 represent possible distributions of the x -component of the perturbed electric field E_x^1 . Curve 2 corresponds to $\cos \delta > 0$ and curve 3 to $\cos \delta < 0$. The stabilizing role of the perturbed field (curve 3) consists of that it causes the ions to move toward the center of the layer.

Instability means disturbance of the balance between the particles leaving the layer and the particles produced by ionization. As a result, an excess number of particles, as compared with the equilibrium state, is formed in the layer. The excess number can be determined by multiplying the right-hand side of (3.1) by $\cos \nu x$ and by integrating it over the volume $2a \times \tau_y \times \tau_z$ (τ_y and τ_z are half-wavelengths in the y - and z -directions). Now, owing to adequacy of the fraction in (2.4) and of the quantity Θ , we can write the instability condition as

$$\begin{aligned}F_2 k_\omega^2 \frac{b_i}{b_e} D_e &< \frac{2}{3} (k_y \rho)^2 [2D_a + (\omega_e \tau_e)^2 (D_a + D_i)], \\ D_i k^2 &< D_a \rho^2 + \frac{b_i}{2} F_1 \kappa |\cos \delta|.\end{aligned}\quad (3.4)$$

In conclusion, we note that the imaginary part of (3.1) gives a result agreeing with (2.5), but in terms of the coefficient κ .

Examination of the continuity equation for the electron component leads to similar results, i.e., to (2.4), (2.5), and (3.4), since

$$\nabla \Gamma_1^e = \nabla \Gamma_1^i,$$

where Γ_1^e and Γ_1^i are the linearized electron and ion diffusion currents,

respectively. The roles of the individual terms in this equation, however, are different than in the continuity equation for ions. This is due to the antiparallel motion of electrons in an electric field and to their motion under the influence of Lorentz forces. Here, we consider only one problem: we clarify the condition under which the field E_x^1 has a distribution similar to curve 3 in Fig. 1.

Figure 2 shows the orientation of the fields in the plasma layer. The electric field induced in the equilibrium state E varies along the x -coordinate (see (1.9)). To the right of the yz -plane, which passes through the point m (right region), this field is oriented along the x -axis; to the left of this plane (left region), it is counter to the x -axis.

Under the influence of the external fields B and E_0 , the electrons drift in the positive direction of the x -axis with the velocity $\omega_e \tau_e b_{e\perp} E_0$. Since the ion-drift velocity is different (in this case, it is zero, because the cyclotron frequency of the ions was ignored), charge separation occurs, and an electric field E_{01} , which coincides with the positive direction of the x -axis, is induced in the layer. Electron drift under the influence of the fields B and E occurs along the y -axis. The electric field E_1 induced as a result of this is oriented along the y -axis in the left region and counter to it in the right region. The field E_1 , together with the magnetic field, causes electron drift along the x -axis, and in the layer an electric field E_2 is induced that is parallel to the x -axis in the left region and antiparallel in the right region. Thus, the x -component of the induced field is $i(E_{01} + E_2)$ in the left region and $i(E_{01} - E_2)$ in the right region. Hence, it follows that the perturbed field E_x^1 can be distributed similarly to curve 3 (Fig. 1) if in the right

region $E_2 > E_{01}$. Within this region this is equivalent to the requirement $E_1 > E_0$ or $\omega_e \tau_e E > E_0$.

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